Letter to Mork:

“The class has spent numerous frustrating hours working with a few simple problems, searching for a universal connection among it all. Towers, tunnels, grid walks, binomial expansion, and Pascal’s Triangle, all blurred together, with the class at the breaking point of sanity. Then, finally, Jim had an epiphany. The answer was, not surprisingly, quite simple. In every problem there were two variables. It is this simple rule that is the basis of the entire lesson. With the towers there were two colors----* an important point to mention right up front is the fact that the grid problem turned sideways is Pascal’s Triangle; this fact comes into play several times in the analysis. Powers of 2 were seen in the tower problem, in the grid walks, and in Pascal’s Triangle.

In the tower problem, we learned that to find how many possible solutions could be found using 2 colors and varying heights, one could simply use the powers of 2 to find the answer. Simply take the number 2 raised to the power of the number high the towers are. For example, to find how many towers would be in a group of towers 2 high, take 2 to the 2nd power and you get 4; therefore, there are 4 possible solutions.

Another example is to find how many towers would be in a group of towers 3 high, take 2 to the 3rd power and you get 8. The important factor here is that there are exactly two different colors. In the grid walk problem, by adding the totals of the numbers in the diagonals, one would get the numbers 1, 2, 4, 8, ...These numbers are the powers of 2 as well (2^0 = 1; 2^1 = 2; 2 ^ 2 = 4; etc.). As was stated before, the grid walk design is numerically exactly the same as Pascal’s Triangle, so the numbers in each row of the triangle (instead of the diagonals like in the grid problem) added together equal the powers of 2 as well....

The grid walk problem helped us explain why exactly Pascal’s Triangle works. The point was to find out how many possible paths there were to get to the dots; we then wrote that number in the dot. If a dot had the number 10 on it, that means there were 10 different paths to get to that single dot and to get to dot 2 there are 10 different paths.

The tunnel problem was quite different from the others. There were two paths to go through, which again gives us the two choices, but then an additional restriction was added—each path possible had to go through exactly 2 black tunnels and 2 white tunnels. After finding 6 possible paths, and examining our past problems, we found some connection linking this problem to the others. Looking at row 4 in Pascal’s Triangle and diagonal 4 in the grid walks, we see the numbers 1 4 6 4 1. The number 6 in the middle is how many possible paths there w2ere, the number 4’s on either side represent how many total stations there were (2 white + 2 black), and the number 1’s represent the colors (the number 1 is on each side: 1 + 1 = 2: there are two colors).

An extremely important part that I forgot to mention earlier with binomial expansion is the significance of the coefficients and the variables in connection with the towers. By using Pascal’s Triangle to find the coefficients (row III) of (w + b)3, we know that they are 1 3 3 1. By adding in the variables, we see that the answer is 1 w^3+ $3w^2b$ + $3wb^2$+ b^3. Looking at this problem term by term, we can apply this to the towers and say that there is 1 possible combination with all whites, 3 possible combinations with 1 white and 2 blacks, 3 possible combinations with 1 black and 2 whites, and 1 possible combination with 3 blacks."

Mid-term self-evaluation

"Looking back on the first half of the semester, I am quite proud of how far I've come in this class. I entered this class with a mixture of opinions, ranging from "this class is going to be extremely difficult because I've often struggled at math" to "this class is going to be extremely easy; after all, it's math for elementary school teachers—how hard can it be." I've found that this class follows that funny little rule of "it's simple, but not easy." I constantly find myself challenged to approach problems in ways I'd never through of before. And then once I'd figured a method out, I'd have to verbalize it, which proved to be the greatest challenge by far. Looking back at the very first problem we dealt with—the tower building, I kind of chuckle to myself at how I approached the write-up. I clearly remember staring at the questions "provide a convincing argument that you have found all possibilities," and "is the order in which you produced the towers important"?

For my entire life, it's been "here's a formula"—good luck. I never knew how wrong we were. It is so amazing to me that math actually has depth. I never thought I would be able to comprehend it. But I do. I'm actually learning in this class—learning that binomial expansion, something I worked with all my freshman year in high school but never fully grasped, means using two variables. Such a simple concept, but not at all easy for me to understand. Recently we've been working with alternate number systems. The impact this has had on me is indescribable. What a wonderful way for us to change our perspectives, forget what we've already learned, and look at it from a child's perspective." As we move through the last half of the semester, I approach this class with a renewed sense of anticipation. I did not possess that first day of class. By being open-minded and flexible, I hope to gain as much from this class as possible. And I know that once I become a teacher, I will never forget this class and the invaluable lessons I have learned."

Final self-evaluation

"I know that for the rest of my life I will remember my first semester. I will remember these few months because of the math course simply titled, Math for Elementary Teachers. The name seems somewhat ambiguous. Myself and others wondered that first day what exactly we were getting ourselves into. I do not think anyone of us was prepared for the lessons we learned....This class proved to be far more challenging than anything we could have imagined. The main point of this class, the lesson that I will take with me forever as a teacher, is how we were taught to forget everything we had been taught and go through the processes as a child would. For me, this opened up a whole new perspective as I started from scratch, letting go of my previous knowledge. I've found that this class follows that funny little rule of "it's simple, but not easy." I constantly find myself challenged to approach problems in ways I'd never through of before. And then once I'd figured a method out, I'd have to verbalize it, which proved to be the greatest challenge by far.

The time it took the class to realize the connection between all of our different problems: the towers, the tunnels, Pascal's Triangle, and the grid walks, reveals how confused our own processes of learning had been. If we were given those problems today, I am willing to bet that just about every one of us would make the connections inside of a week's time, most likely sooner. But at the beginning of the class, as we all came in with our preconceived notions of mathematics as simply finding a formula and getting the right answer, we struggled to allow our minds to work in a way to truly understand the overall connections. I find it both amusing and sad that although I used Pascal's triangle many times throughout high school, I never truly understood why it worked. By completing the grid walk exercises, so many things have become clear to me. Well, I learned the secret of the grid walks and of Pascal's Triangle. The two numbers directly above any given number represent the number of ways to get to that certain dot. I really wish I had known this when I was in Algebra. all we were taught then was that, for some reason unknown to me and probably many others in the class, the rows of Pascal's Triangle made up the coefficients in binomial expansion. Now I not only understand how to use the triangle algebraically, I know why it works. The numbers in an algebraic expression along with the variables can represent different groups of objects, in a manner of speaking. It all makes sense. This simple understanding, but by no means easy, is so rewarding because the knowledge has become my own. And I know that ten years from now, i will still be able to explain this problem.

Works written by Richard R. Skemp and Liping Ma address the notion that learning mathematics is not just about learning formulas and plugging in numbers. They illustrate the importance of developing a thorough understanding of mathematical concepts—the how and whys of problem solving. When children are given only a process and not a true explanation of material, it is the children who will suffer. They need to fully comprehend what is the underlying meaning behind math concepts so they can make it their own knowledge. The simple idea is that once the children make it their own knowledge, once they can explain it themselves, it will be theirs forever.

It was the best of times; it was the worst of times. I think many students who have taken Math for Elementary Teachers will agree that this quote sums up the first semester. We all struggled to forget our own preconceptions of teaching mathematics by examining the learning process. But with this struggle came the crucial lessons that are forever cemented in our minds. Not only do children need to be allowed to learn at their own pace, not only do we as teachers need to treat each child as an individual, but most importantly, it is up to us as elementary school teachers to help them understand the importance and reasoning of mathematical understanding of the root of the concept."