Learning and Individual Differences 20 (2010) 110–122

Contents lists available at ScienceDirect

Learning and Individual Differences

journal homepage: www.elsevier.com/locate/lindif

Working memory and mathematics: A review of developmental, individual difference, and cognitive approaches

Kimberly P. Raghubar [a], Marcia A. Barnes [b,*], Steven A. Hecht [b]

[a] Department of Psychology, 126 Heyne Building, University of Houston, Houston, TX 77204, USA
[b] University of Texas-Houston Health Science Center, 7000 Fannin, Suite 2401, Houston, TX 77030, USA

ARTICLE INFO

Article history:
Received 19 February 2009
Received in revised form 15 October 2009
Accepted 15 October 2009

Keywords:
Working memory
Mathematical processing
Individual differences
Development
Dual task studies

ABSTRACT

Working memory refers to a mental workspace, involved in controlling, regulating, and actively maintaining relevant information to accomplish complex cognitive tasks (e.g. mathematical processing). Despite the potential relevance of a relation between working memory and math for understanding developmental and individual differences in mathematical skills, the nature of this relationship is not well-understood. This paper reviews four approaches that address the relation of working memory and math: 1) dual task studies establishing the role of working memory during on-line math performance; 2) individual difference studies examining working memory in children with math difficulties; 3) studies of working memory as a predictor of mathematical outcomes; and 4) longitudinal studies of working memory and math. The goal of this review is to evaluate current information on the nature of the relationship between working memory and math provided by these four approaches, and to present some of the outstanding questions for future research.
© 2009 Elsevier Inc. All rights reserved.

In their 2005 review LeFevre, DeStefano, Coleman and Shanahan (2005) noted that although a connection between working memory and mathematical processing has long been proposed the evidence connecting the two is relatively sparse. Although considerable controversy remains about whether general purpose cognitive processes such as working memory are *causally* implicated in mathematical development and disabilities (Geary, Hoard, Nugent & Byrd-Craven, 2007 versus Butterworth & Reigosa, 2007), many recent studies support the notion that working memory is related to and important for performance on mathematical tasks. This paper reviews evidence from four types of studies to evaluate the proposed relation of working memory and mathematics: 1) experimental dual task studies that test whether specific working memory resources are brought to bear during mathematical processing; 2) individual difference studies that ask whether working memory problems differentiate children with difficulties in math from their typically achieving peers; 3) studies that address whether and how working memory is related to specific mathematical outcomes and processes in children of different ages and abilities; and 4) longitudinal studies that ask whether the development of working memory is related to growth in mathematical skills. This review evaluates evidence on the relation of working memory and mathematics provided from these four approaches, and it presents outstanding questions for future research with a particular emphasis on the application of research to understanding typical and atypical development of mathematical skills.

What is the basis for proposing that working memory and mathematical processing are related? Working memory refers to a mental workspace that is involved in controlling, regulating, and actively maintaining relevant information to accomplish complex cognitive tasks (Miyake & Shah, 1999). Mathematical competence entails a variety of complex skills that encompass somewhat different conceptual content and procedures (e.g., arithmetic, algebra, and geometry); problem solving in these domains often involves the holding of partial information and the processing of new information to arrive at a solution, which ought to require working memory resources. This description applies to both early informal mathematical problem solving in preschool children (Bisanz, Sherman, Rasmussen, & Ho, 2005) and complex mathematical tasks in older children and adults, such as multi-step procedures in multi-digit arithmetic, word problem solving, and numerical estimation (Kintsch & Greeno, 1985; Siegler & Booth, 2005; Swanson, 2004; Tronsky, 2005). Furthermore, very young children may touch and point to objects to compensate for their developmentally-limited working memory skills, which are necessary for representing and updating quantitative information (Alibali & DiRusso, 1999). Thus, the very nature of many mathematical tasks would seem to require or at least be supported by working memory, and this may be relevant when thinking about both typical and atypical development of mathematical skills.

Acknowledged relations between working memory and math have both theoretical and practical relevance. Working memory is not a component of the main theories of mathematical cognition [e.g. Abstract code model (McCloskey, 1992); Triple-code model (Dehaene, 1992); Encoding complex model (Campbell & Clarke, 1992)], though some

* Corresponding author. Tel.: +1 713 500 3826; fax: +1 713 500 3878.
E-mail address: Marcia.barnes@uth.tmc.edu (M.A. Barnes).

1041-6080/$ – see front matter © 2009 Elsevier Inc. All rights reserved.
doi:10.1016/j.lindif.2009.10.005

conceptual frameworks for mathematical development and disability do include this construct (e.g., Geary & Hoard, 2005). As such, these models may be missing an explanatory construct for understanding not only mathematical performance and individual differences but also the development and operation of mathematical skills across the life span (Duverne, Lemaire, & Vandierendonck, 2008; Hecht, 2002; LeFevre et al., 2005). From an applied perspective, knowing whether working memory is related to how children learn mathematics and why some children have difficulties in math may be important for instructional design. For example, some instructional methods allow young children to learn new mathematical concepts and procedures by taking the working memory demands of mathematical tasks and the strategies children employ in their problem solving into account (e.g., Case & Okamoto, 1996).

We provide a brief review of working memory models prior to discussing the relation of working memory to math. Working memory models vary on a number of dimensions, one of which is whether working memory is a multi-component system composed of subsystems specialized to handle different kinds of information (e.g., Baddeley & Hitch, 1974) or is a unitary system that is primarily involved in attentional control (e.g., Engle, Tuholski, Laughlin, & Conway, 1999). Baddeley's (Baddeley & Hitch, 1974; Baddeley & Logie, 1999) multi-component model describes working memory as a limited capacity central executive system that interacts with a set of two passive subsystems used for the temporary storage of different classes of information: the speech-based phonological loop and the visual–spatial sketchpad. The phonological loop is responsible for the temporary storage of verbal information; items are held in a phonological store of limited duration, and the items are maintained within the store via the process of articulation. The visual–spatial sketchpad is responsible for the storage of visual–spatial information over brief periods, and it also plays a key role in the generation and manipulation of mental images. Both the phonological loop and the visual–spatial sketchpad are in direct contact with the central executive, which is considered to be primarily responsible for the coordinating activity within the cognitive system, but it also devotes some of its resources to increasing the amount of information that can be held in the two passive subsystems. This multi-componential approach allows one to ask questions about the mental code that is being used in mathematical tasks and whether such codes vary as a function of the specific mathematical task, skill-level (developmental and individual differences), and strategy use (LeFevre et al., 2005).

Studies will be reviewed that use this approach to the measurement of working memory in relation to math. These studies are of two types: in experimental dual task studies, participants perform tasks that draw on the phonological loop, visual–spatial sketchpad, or central executive resources at the same time as they perform math tasks. In individual difference and developmental studies, performance on measures tapping the phonological loop, the visual–spatial sketchpad, and the central executive are considered in relation to mathematical ability/disability or performance on specific mathematical tasks. In this second type of study, working memory measures (requiring concurrent storage and processing) are frequently used to index the central executive capacity of individuals and they are sometimes contrasted with performance on short-term memory tasks in which participants are required to hold small amounts of verbal or visual–spatial information passively, and then reproduce the information in a sequential or untransformed fashion (tapping the phonological loop and the visual–spatial sketchpad, respectively).

According to a single-capacity or domain general view of working memory (Engle et al., 1999), variations in working memory across individuals reflect the capacity of the central executive. Processes that have been attributed to the central executive include inhibition of irrelevant information, task switching, information updating, goal management, and strategic retrieval from long term memory. In this view, working memory capacity has more to do with the ability to control and allocate attention during complex cognitive tasks than with the amount of information that can be stored. Greater working capacity results from greater attentional control. It is worth noting that most measures of working memory likely require some combination of these processes. However, studies that specifically relate the development of these somewhat distinct executive processes (e.g., inhibition of irrelevant information, updating, and attention switching) to mathematical outcomes are also considered in this review.

1. Experimental investigations of working memory and mathematical processing

The role of working memory in simple arithmetic or in any particular cognitive task can be directly examined during on-line performance using dual task methods. Dual task experiments involve the performance of a criterion task (e.g., solving simple arithmetic problems) while simultaneously performing a secondary task (e.g., articulating syllables). Secondary tasks are chosen to represent different components of the proposed working memory system; for example, verbal tasks, such as articulating "the the the", tax the phonological loop (e.g., De Rammelaere, Stuyven, & Vandierendonck, 1999; Hecht, 2002) whereas visual–spatial tasks, such as spatial finger tapping (e.g., Seitz & Schumann-Hengsteler, 2000), tax the visual–spatial sketchpad. Performance when the criterion and secondary tasks are combined is compared to performance when the criterion task is completed alone. If the criterion and secondary tasks use overlapping cognitive resources then performance on the criterion task will get worse as the secondary task becomes more demanding. It has been argued that dual task studies provide the most compelling evidence for the specific processes involved in the task of interest because they can be used to isolate the roles of the different components of working memory. This approach has been predominantly used with typically achieving adults in the investigation of single-digit and multi-digit arithmetic. We review these studies as well as the few dual task studies in children.

1.1. Adult studies of single-digit arithmetic

A central executive load interferes with the solution of single-digit problems across operations (Ashcraft, Donley, Halas, & Vakali, 1992; De Rammelaere et al., 1999; De Rammelaere, Stuyven, & Vandierendonck, 2001; Hecht, 2002; Lemaire, Abdi, & Fayol, 1996; Seitz & Schumann-Hengsteler, 2000, 2002). In contrast, whether the phonological loop or visual–spatial sketchpad is involved in single-digit calculation depends on the size of the problem, the mathematical operation involved, how single-digit arithmetic is learned (DeStefano & LeFevre, 2004), and strategy selection (Hecht, 2002).

The evidence for the role of the phonological loop in addition and subtraction appears to depend, in large part, not so much on the operation under consideration, but the strategy used to complete the computation. Studies that have looked at strategy use report that phonological load interferes with performance on trials where counting strategies are employed (Hecht, 2002; Imbo & Vandierendonck, 2007a). Whether or not phonological load impairs performance on subtraction problems also likely depends on strategy use, such as counting down or transformation (Imbo & Vandierendonck, 2007a; Seyler, Kirk, & Ashcraft, 2003). For the most part, phonological interference has not been demonstrated to impair performance on single-digit multiplication problems, particularly for easy multiplication problems (i.e., both operands less than five) (De Rammelaere et al., 2001; Seitz & Schumann-Hengsteler, 2000, 2002), consistent with the notion that access to permanent information in long-term memory does not require intervention by the working memory slave systems (Baddeley, 1990, 1996). However, phonological load does interfere with the performance of single-digit multiplication in Korean speaking university students

(Lee & Kang, 2002), who may rely on phonological codes when storing and accessing multiplication facts compared to individuals taught in other languages (e.g. English and German) (DeStefano & LeFevre, 2004). Related research suggests that Chinese-speaking individuals store and access multiplication facts using phonological codes because of the structure of their language for number and because of educational factors related to how multiplication facts are instructed (LeFevre, Lei, Smith-Chant, & Mullins, 2001; LeFevre & Liu, 1997).

There is a paucity of research examining the role of the visual–spatial sketchpad in single-digit calculation. Visual–spatial interference has been shown to disrupt performance on subtraction (Lee & Kang, 2002), but not multiplication (Lee & Kang, 2002; Seitz & Schumann-Hengsteler, 2000). Findings may depend on factors such as the nature of the secondary task; for example, visual–spatial as opposed to static visual secondary tasks could produce different findings to the extent that the mental number line that may underlie aspects of mathematical performance is thought to be an abstract spatial rather than concrete visual representation (Knops, Thirion, Hubbard, Michel, & Dehaene, 2009; Zorzi, Priftis, & Umilta, 2002). As well, participants' adherence to secondary task instructions could affect findings; unlike verbal secondary tasks that are heard by the examiner, visual–spatial secondary tasks are often done out of the examiner's sight (Raghubar, 2009).

1.2. Adult studies of multi-digit arithmetic

Similar to single-digit calculation, multi-digit calculation requires access to simple arithmetic facts; however, it also necessitates procedures of incrementing (carrying) or decrementing (borrowing) across numerical columns. In particular, multi-digit arithmetic involves maintenance of intermediate sums and management of carry demands, particularly in dual task studies because multi-digit arithmetic is done mentally rather than using pencil and paper. Studies conducted over the past decade suggest that presentation variables, such as auditory versus visual presentation or horizontal versus vertical presentation format, and ensuing strategy use influence which components of working memory are recruited in problems requiring or not requiring regrouping (i.e., carrying and borrowing). Many studies suggest that the maintenance of intermediate results requires resources from both the central executive and the phonological loop (reviewed in DeStefano & LeFevre, 2004). Less work has been done on the visual–spatial sketchpad. What data there are suggest that the phonological loop may be involved in multi-digit arithmetic when problems are presented in either auditory or visual format because individuals may translate the visually presented information into a phonological code for temporary storage (Noel, Desert, Aubrun, & Seron, 2001). The visual–spatial sketchpad may be recruited only when problems are presented visually (e.g., Logie, Gilhooly, & Wynn, 1994, but see Seitz & Schumann-Hengsteler, 2000 Exp 1).

The central executive is consistently found to be important for the carry operation in addition (Imbo, Vandierendonck, & De Rammelaere, 2007; Seitz & Schumann-Hengsteler, 2002, but see limited role for phonological loop in Furst & Hitch, 2000: Exp 2), and in complex multiplication (Imbo, Vandierendonck, & Vergauwe, 2008; Seitz & Schumann-Hengsteler, 2000; see limited role for phonological loop in Seitz & Schumann-Hengsteler, 2002). An increasing role of the central executive is also implicated across operations as the number of carry or borrow operations increases and with higher values of the carry (Imbo et al., 2007, 2008).

In summary, findings from the adult dual task literature highlight the importance of attending to several factors in order to understand the connections between working memory and mathematical performance. In terms of the working memory measures themselves this includes attending to what the task is measuring (e.g., visual or spatial interference). In terms of the mathematical tasks this includes knowing something about problem complexity, instructional and linguistic factors, and presentation formats, all of which can affect strategy-use.

1.3. Dual task studies with children

Although dual task research has primarily been conducted with adults, two studies have investigated the role of working memory in children's on-line arithmetic performance. McKenzie, Bull, and Gray (2003) examined the importance of phonological and visual–spatial codes in simple arithmetic performance at different ages. Younger (6–7 years old) and older (8–9 years old) children heard arithmetic problems (containing two or three addends; e.g., $5+7$ or $5+7+8$), under three conditions: baseline, phonological interference (recording of a children's story read in Norwegian), and visual–spatial interference (screen displaying a matrix of black and white squares which randomly change from black to white and vice versa). The younger children remained largely unaffected by phonological interference, yet their performance was severely impaired by visual–spatial interference (even though the problems were presented aurally) whereas the older children were affected by phonological interference as well as visual–spatial interference, though not to the same extent as the younger children. The younger children may have been relying almost solely on visual–spatial strategies, but the older children were likely using a mix of strategies: a primarily verbal approach supplemented by visual–spatial resources.

In order to examine the contribution of executive resources to simple arithmetic, including its role in strategy use, Imbo and Vandierendonck (2007b) assessed children's (4th to 6th grades) solution strategies to large sum single-digit addition problems (e.g., $8+7$) under five conditions: naming (reading the provided answer from the computer screen); choice (choosing their own strategies and reporting the strategy used to get the answer for each problem); and three no-choice conditions (where they had to use only one strategy for all problems as provided by the experimenter — retrieval, decomposition, or transformation). Children solved the addition problems under these five conditions with and without a dual task requirement, which loaded the executive component of working memory. The dual task was a continuous choice reaction time (CRT) task involving presentation of low and high tones at varying intervals (e.g. 2000 or 2500 ms) for which participants pressed a computer key for high tones and another key for low tones.

Children required executive working memory resources to solve large sum addition problems: there was an effect of the dual task even in the condition where the answer simply had to be named, though the effect was larger when children had to retrieve the answer from long-term memory themselves. Of interest are the findings relating working memory to strategy use. Working memory load did not appear to influence strategy selection in the choice condition; that is, children did not change the mix of strategies they brought to bear in solving the problems in the dual task versus the no dual task conditions. Developmentally, the effect of working memory load decreased linearly or showed a tendency to decrease across the grades on response times for problems solved using retrieval and counting strategies, but not transformation strategies. Therefore, as children grow older and are presumably more experienced with solving addition problems, they become more efficient in the execution of retrieval and counting strategies resulting in reduced need for executive resources.

These child dual task studies suggest that central executive resources are implicated in children's arithmetic performance and the amount of resources recruited varies with strategy use, much like it does for adults, as well as with age or experience. It also appears that visual–spatial working memory may be implicated to a greater extent in the math performance of younger children who are in the process of acquiring basic arithmetic skills (also see Rourke, 1993), but that with experience, verbal working memory comes to support arithmetic performance to a greater extent.

1.4. Conclusions

What does the extensive dual task literature in single- and multi-digit arithmetic in adults and the few studies of dual task studies in children tell us about the relation of working memory and math? First, working memory is involved in mathematical performance whether it is single- or multi-digit arithmetic. Second, more is known about the role of the phonological loop in arithmetic performance than is known about the role of the visual–spatial sketchpad. Third, task variables related to both the secondary working memory task (e.g., visual or spatial, whether and how the task is being carried out) and the primary mathematical task (e.g., type and size of operation, presentation format, and other factors that affect strategy use) are critically important for understanding the relation of working memory and math. Interestingly, neuroimaging studies show that factors, such as presentation format and presentation rate, affect how the brain processes mathematical information (Simon & Rivera, 2007), which provides converging evidence for the importance of task- and strategy-related variables in mathematical cognition. Fourth, sources of individual variation whether they be age/experience, strategy use, or instructional and language factors may determine the ways in which different components of working memory and mathematical performance are related. These are also recurring themes in the developmental and individual difference studies reviewed below.

What do these studies *not* tell us about the relation of working memory and math? In contrast to many developmental and individual difference studies, dual task studies pay a great deal of attention to the nature of the working memory task (i.e., none of the secondary tasks involve number, load may be systematically manipulated) and to the math measures (i.e., separation of tasks by arithmetic operation, attention paid to problem size and strategies). Experimentally, these are strengths of the dual task design. However, the mathematical tasks that children encounter in everyday math learning and in studies that relate working memory and math are not of this nature. For example, most measures of single- and multi-digit arithmetic in child studies use mixed operation problem sets. Why might this be important? Different resources may be recruited by different task-related factors such as single versus mixed operation sets, with the mixed sets possibly requiring the ability to monitor and switch between operations. Similarly, multi-digit arithmetic problems are not often done using mental arithmetic as occurs in dual task experiments. Pencil and paper arithmetic likely reduces, though does not eliminate, demands on working memory by allowing the transcription of partial results. However, there are many situations in which mental arithmetic is used in everyday contexts, though approximate as opposed to exact arithmetic may characterize many of these situations. In this regard, Kalaman and LeFevre (2007) have recently found that in adults, verbal working memory is implicated in both exact and approximate arithmetic (addition of two-digit numbers), but plays a greater role in exact than approximate arithmetic. Such findings begin to take into account the types of mathematical tasks that individuals are called upon to use in everyday situations and may be relevant when applied to studies of mathematical development and disability.

2. Working memory in children with math difficulties

Individual difference studies are commonly found in the child and disability literatures and to a much lesser extent in the adult cognitive literature. The next section reviews individual difference studies that compare working memory performance in children with and without difficulties in math. These studies address the relation of working memory and math by asking whether children at the lower ends of the distribution in math have co-occurring deficits in working memory. It has recently been argued that the severity of children's difficulties in math may be important for understanding underlying cognitive weaknesses that may impact their mathematical performance (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Mazzocco, 2007; but see Fletcher, Lyon, Fuchs, & Barnes, 2007 for a dimensional view of mathematical abilities and disabilities). We make note of studies where severity seems to matter for interpreting the relation of working memory and math, but otherwise do not distinguish between studies with more and less stringent cut-offs for determining mathematical disability/difficulty. The evidence for verbal working memory deficits in children with difficulties in math will be discussed followed by the evidence for deficits in visual–spatial working memory. Readers are referred to Table 1 for details on the samples and methods used in these studies.

2.1. Verbal working memory and math difficulties

A recent meta-analysis of 28 studies comparing the cognitive characteristics of children with and without math difficulties suggests that differences in verbal working memory characterize children with math difficulties after controlling for effects of several other variables, such as age, IQ, naming speed, and short-term memory for words and digits (Swanson & Jerman, 2006). However, the measures of verbal working memory often required the use of number information because some studies combine verbal and numerical working memory tasks into a composite measure of the central executive (see Geary, Hoard, Byrd-Craven et al., 2007; Geary, Hoard, Nugent, & Byrd-Craven, 2008) and other studies use *only* numerical tasks as their indicator of verbal working memory (e.g., Geary, Hamson, & Hoard, 2000; Geary, Hoard, & Hamson, 1999; Landerl, Bevan, & Butterworth, 2004; Wu et al., 2008). Although working memory tasks that employ numerical information are considered to tap verbal working memory, these tasks have also been referred to as domain-specific (i.e., number-specific) measures of working memory (LeFevre et al., 2005). The two most commonly used numerical working memory tasks are digit span backward, whereby participants are presented with a sequence of numbers and are asked to reproduce the sequence in the reverse order of presentation; and counting span, which involves remembering numbers of counted objects, for example, counting the yellow dots in a set of yellow and blue dots for a specified number of sets (typically 2–5), and then recalling the counts for each set in the correct order. The assumption that numerical working memory tasks are de facto measures of verbal working memory and that different numerical working memory tasks provide similar measures of verbal working memory deserves some scrutiny as discussed below.

Numerical measures of verbal working memory are more frequently related to math difficulties than are non-numerical measures of working memory. Studies that have separated verbal working memory measures without numerical information (e.g., word span backward) from those involving number (e.g., digit span backward) have found that the latter, but not the former distinguish children with difficulties in math (Passolunghi & Cornoldi, 2008; Passolunghi & Siegel, 2001, 2004). Furthermore, studies employing verbal working memory tasks, such as the listening span task (with concurrent language comprehension and recall) that likely require greater central executive resources than backward word span or digit span tasks do not consistently differentiate between children with and without math difficulties (Reukhala, 2001; Siegel & Ryan, 1989; Swanson & Beebe-Frankenberger, 2004; van der Sluis, van der Leij, & de Jong, 2005 vs. D'Amico & Guarnera, 2005; Fuchs et al., 2008; Passolunghi & Cornoldi, 2008). The stronger relation between numerical versus non-numerical measures of verbal working memory and math is consistent with studies demonstrating the

Table 1
The samples and measures of working memory and math ability studies.

Study	Type	Control	Verbal WM	Visual–spatial WM	Math outcome
Andersson and Lyxell (2007)	Child, ID	Verbal IQ[a] Nonverbal IQ[a] Age[b] Reading[b]	Animal dual task, Counting span, Trail making (written), Color stroop (inhibition control), Verbal fluency, Number matching (general processing speed), Crossing out (selective attention)	Visual matrix span, Corsi block	Math screening test: Magnitude comparison; number transcoding (verbal to Arabic); multi-digit addition and subtraction; written addition problems; missing number sequence
Berg (2008)	Child, Predictor	Age[c] Reading[c] Processing speed[c] Short term memory[c]	Semantic association, Digit sentence span (Composite)	Visual matrix span, Corsi block (Composite)	WRAT-3 Arithmetic
Bull et al. (1999)	Child, ID	IQ[a] Reading[a]	Wisconsin card sort task	Corsi block	Group mathematics test: Word problems (read by experimenter); single- and multi-digit addition and subtraction
D'Amico and Guarnera (2005)	Child, ID	Reading[b]	Listening recall, DS backward, Making trails (verbal and color)	Static matrices, Corsi block	ABCA battery (10 subtests) Incl. mental and written calculation, number dictation, number comparison using "<" or ">"
Fuchs et al. (2005)	Child, Predictor	Attention[c] Language[c] Nonverbal problem solving[c] Phonological processing[c] Processing speed[c] Executive function[c] Intervention effects[c]	Listening recall		CBM computation, Addition fact fluency, Subtraction fact fluency, Concepts/applications, Story problems, WJ III Calculation, WJ III Applied problems
Fuchs et al. (2008)	Child, ID		Listening recall, DS backward		Test of computational fluency, Addition fact fluency, Subtraction fact fluency, Simple word problems, Algorithmic word problems, Complex word problems,
Geary et al. (2000)	Child, ID	IQ[a,b] Reading[b]	DS backward		WIAT Math reasoning
Geary et al. (2004)	Child, ID	IQ[a,b]	Counting span		WIAT Math reasoning
Geary, Hoard, Byrd-Craven, et al. (2007)	Child, ID, and Mediator of performance on math cognition tasks	IQ[b]	Listening recall, Counting recall, DS backward (Composite)	Corsi block, Mazes memory (Composite)	WIAT-II-Abbreviated Numerical operations
Geary et al. (1999)	Child, ID	IQ[a,b] Reading[b]	DS backward		WIAT Math reasoning
Geary et al. (2008)	Child, ID	IQ[b]	Listening recall, Counting recall, DS backward (Composite)	Corsi block, Mazes memory (Composite)	WIAT-II-Abbreviated Numerical operations
Hitch and McAuley (1991) (Exp 1)	Child, ID	Nonverbal IQ[b,d] Reading[b,d]	Counting span		Group mathematics test (*see above*)
Holmes and Adams (2006)	Child, Predictor		Listening recall	Mazes memory task	Age-appropriate assessments developed measuring: Number and algebra; Shape, space, and measures; Handling data; Mental arithmetic
Holmes et al. (2008)	Child, Predictor			Static matrices, Corsi block	Number and algebra; Shape, space, and measures; Handling data; Mental arithmetic
Kyttala and Lehto (2008)	Adolescent, Predictor	Nonverbal IQ[c]		Mental rotation, Static matrices, Corsi block	Mathematics test (Finnish)-Total score and Mental arithmetic, Geometry, and Word problems subscores
Landerl et al. (2004)	Child, ID	Nonverbal IQ[b] Reading[b] Age[a]	DS backward		Response latency and accuracy on computerized simple addition, subtraction, and multiplication tasks
Mabbot and Bisanz (2008)	Child, ID		DS backward, Operation span		WRAT-3 Arithmetic
McLean and Hitch (1999)	Child, ID	Reading[b]	Making trails (written, verbal, color; switching), Crossing out (selective attention), Missing item (hold and manipulate info accessed in LTM)	Static matrices, Corsi block	Graded arithmetic-mathematics test
Passolunghi and Cornoldi (2008)	Child, ID	Verbal IQ[d] Age[d] Gender[d]	DS backward, Word span backward, Listening span comp.	Corsi block (forward and backward)	Standardized Italian Arithmetic Battery (AC-MT): Written calculations, e.g., comparisons, decompositions, and sequencing of complex numbers
Passolunghi and Siegel (2001)	Child, ID	Age[d] Gender[d] Reading[b]	Listening recall, Animal dual task, Listening span comp., Counting span, DS backward		Standardized Italian mathematics test: Written arithmetical word problems; manipulation of Arabic and verbal numerals

Table 1 (*continued*)

Study	Type	Control	Verbal WM	Visual–spatial WM	Math outcome
Passolunghi and Siegel (2004)	Child, ID	Age[d] Gender[d] Vocabulary[d] Reading comprehension[b]	Listening recall, Listening span comp., Counting span, Word span backward, DS backward		Standardized Italian mathematics test (see above)
Reukhala (2001)	Adolescent, ID, and Predictor		Listening recall	Mental rotation, Static matrices, Corsi block	Total score on a National mathematical test including both mental arithmetic and paper-and pencil tasks
Rosselli et al. (2006)	Child, ID	Nonverbal IQ[b] Reading[b]	DS backward		WRAT-3 Arithmetic
Siegel and Ryan (1989)	Child, ID	IQ[b] Reading[b]	Listening span comp., Counting span		WRAT Arithmetic
Swanson and Beebe-Frankenberger (2004)	Child, ID, and Predictor of word problem solving ability	Nonverbal IQ[a,c] Algorithm knowledge[c] Math calculation[c] Reading[c] Semantic processing[c] Inhibition[c] Speed[c] Phonological processing[c] Age[c] Short-term memory[c]	Listening recall, Semantic association, Digit sentence span, DS backward	Visual matrix span, Mapping and directions	WISC-III Arithmetic
Temple and Sherwood (2002)	Child, ID	Verbal IQ[d] Age[d]	DS backward	Corsi block	WOND
van der Sluis et al. (2005)	Child, ID	Nonverbal IQ[b] Reading[b]	DS backward, Listening span comp., Counting span	Static matrices, Corsi block	Arithmetic tempo test contains 3 timed subtests: addition, subtraction, and multiplication
Wilson and Swanson (2001)	Child and Adult, Predictor	Age[c] Gender[c] Reading[b,c]	Story retelling, Semantic association (Composite)	Visual matrix span, Mapping and directions (Composite)	WRAT-R Arithmetic
Wu et al. (2008)	Child, ID and Predictor of strategy use in simple arithmetic	IQ[b]	Counting span, DS backward	Corsi block	WIAT Math composite

Note: ID = Individual difference; DS backward = Digit span backward; Listening span comp. = Listening span completion; WIAT = Wechsler Individual Achievement Test; WISC-III = Wechsler Intelligence Scale for Children —Third Edition; WOND = Wechsler Objective Numerical Dimensions; WRAT = Wide Range Achievement Test; WJ-III = Woodcock Johnson III — Tests of Achievement.

[a] Variables controlled for in group comparisons (e.g., controlling for age in ANCOVA).

[b] Scores had to meet inclusion criteria (e.g. IQ > 80), scores were used for group assignment (e.g. reading scores used for group assignment to MD or MD + RD), or scores between groups were tested for differences.

[c] Variables accounted for in regression equations.

[d] Groups matched on variables.

importance of domain specific knowledge to general cognitive performance including memory (review in Bjorklund, 2005).

Math difficulties are more consistently predicted by counting span versus backward digit span. In several studies, counting span distinguishes children with math difficulties from their typically achieving peers (Andersson & Lyxell, 2007; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Hitch & McAuley, 1991; Passolunghi & Siegel, 2001, 2004; Siegel & Ryan, 1989; Wu et al., 2008). The findings are not as consistent for digit span backward: several studies report null findings (Geary et al., 2000; Landerl et al., 2004; Temple & Sherwood, 2002; van der Sluis et al., 2005); some studies find differences between groups of children with math difficulties and typically achieving children (D'Amico & Guarnera, 2005; Fuchs et al., 2008; Passolunghi & Cornoldi, 2008; Passolunghi & Siegel, 2001, 2004; Rosselli, Matute, Pinto, & Ardila, 2006; Swanson & Beebe-Frankenberger, 2004; Wu et al., 2008); and yet other studies find that digit span backward specifically distinguishes children with severe math difficulties from both children with less severe math difficulties, and children with typical achievement in math (Mabbot & Bisanz, 2008).

Digit span backward tasks can be performed using non-verbal strategies. Although most researchers agree that digit span backward is a measure of simultaneous storage and transformation, in what form is the relevant information stored and how is it transformed? There does not appear to be a definitive answer to this question, as strategy use is likely to be important. Digit span backward is most often described as a measure of verbal working memory, involving both the phonological loop and the central executive, and in some studies digit span backward does show effects of phonological similarity (Rosen & Engle, 1997). However, another possibility articulated most recently by Berch (2008) is that visual–spatial representations may play a role in digit span backward performance. This may be particularly true for younger children as developmental studies suggest there is less separation of the verbal and visual–spatial working memory systems before 8 years of age (Hale, Bronik, & Fry, 1997). Neuropsychological studies lend support for the role of visual–spatial representations in digit span backward. Positron emission tomography (PET) studies have shown that visual processing areas are activated during digit span backward, even when participants were blindfolded so that activations could not be attributed to visual-processing per se (Gerton et al., 2004). Similarly, a series of cognitive experiments by Li and Lewandowsky (1995) suggest that visual processes are implicated in backward recall but not in forward recall of information. While it is commonplace to ascribe verbal strategies to tasks designed to tap visual–spatial processes, these findings suggest that the reverse may also be true: Overtly verbal tasks can be approached using visual–spatial strategies, so visual–spatial processing may contribute to performance on digit span backward.

Without knowing how the digit span backward task is being performed by individuals, it is difficult to say exactly what aspect(s) of working memory this task is measuring.

Digit span backward might draw to greater or lesser extents on central executive processes related to differences in number words across languages. It is interesting to note that studies conducted with Italian- or Spanish-speaking children more consistently report that children with math difficulties perform worse on digit span backward than same-aged peers (D'Amico & Guarnera, 2005; Passolunghi & Cornoldi, 2008; Passolunghi & Siegel 2001, 2004; Rosselli, Matute, Pinto and Ardila, 2006); the findings for English- and Dutch-speakers are often negative (Geary et al, 1999, 2000; Landerl et al., 2004; Temple & Sherwood, 2002; van der Sluis et al., 2005; but see Mabbot & Bisanz, 2008). In contrast to English and Dutch, the majority of Italian and Spanish number words from one to nine have longer word lengths because they are not monosyllabic. Accordingly, the Spanish and Italian numbers take longer to articulate. Word length and articulation rate are strongly related to memory span in children (Hulme, Thomson, Muir, & Lawrence, 1984). Italian and Spanish children using verbal strategies may require greater executive resources to supplement the storage and rehearsal processes of the phonological loop, or greater controlled attention. Alternatively, they may use visual–spatial strategies when completing digit span backward given the task demands. In any event, the underlying mechanisms and processes used on digit span backward tasks may vary across languages.

In summary, the question of whether verbal working memory serves to distinguish children with and without mathematical difficulties is complicated by several factors. Measures of verbal working memory that use number may be more strongly related to math difficulties than measures that do not use number because of their domain-specificity; that is, these working memory materials draw from the same domain in which the child is impaired. This is not a commentary on the usefulness of numerical working memory tasks in math research, but rather an observation about how such studies that show relations between numerical working memory and mathematical disability might be interpreted. Even when only domain specific working memory tasks are considered several additional issues need to be examined that may affect the interpretation of studies relating these measures of working memory to math difficulties. These factors include: the type of numerical information that needs to be manipulated in the working memory task; the strategies that children bring to bear on these working memory tasks, possibly as a function of age; the severity of the difficulties in math; and the processing requirements of number words in different languages.

2.2. Visual–spatial working memory and math difficulties

Visual–spatial working memory tasks have been employed less often than verbal or numerical working memory tasks in studies of children and math disability. Visual–spatial working memory and the visual–spatial sketchpad are sometimes subdivided according to storage of *static* visual information, such as information about form and color, and storage of *dynamic* visual information, such as information about motion, location, and direction. These divisions reflect those in the literature over whether the critical distinction in the nonverbal domain is that between visual working memory and visual–spatial working memory (paralleling neuroimaging studies that make the distinction between the processing of object-based or location-based information, e.g., Baker, Frith, Frackowiak, & Dolan, 1996) or between passive and active aspects of visual–spatial working memory, the former requiring the recall of visual–spatial information as presented, and the latter requiring some manipulation of that information such as backwards recall (Cornoldi, Rigoni, Venneri, & Vecchi, 2000).

Static and dynamic tasks can vary greatly between studies. Common static visual tasks include the static matrices task and a more demanding variant, the visual matrix span task. In static matrices tasks, participants are asked to remember the location of the black target blocks in a display in which half the blocks are black and half are white. The visual matrix span task is similar to the static matrices task, however, following the display, participants are asked a process question (e.g. Were there any target blocks in the first column?), and are then asked to recall the location of the target blocks. Dynamic visual–spatial tasks include dynamic matrices and Corsi block tasks. In dynamic matrices, the blocks in an empty matrix sequentially flash on and off, and participants recall in correct order the location of the blocks that had flashed. In Corsi block tasks, a series of block taps must be replicated in sequence. Yet other studies employ both a forward and a backward version of the Corsi blocks: the forward version is said to measure passive visual–spatial working memory and the backward version is said to measure active visual–spatial working memory (Passolunghi & Cornoldi, 2008).

Studies employing visual or visual–spatial memory measures differ from each other in a number of ways and these differences between studies make it difficult to draw conclusions across the relatively small number of studies. These studies vary with respect to: the age of the participants; how math difficulties are identified (e.g., Temple & Sherwood, 2002 versus van der Sluis et al., 2005), including the severity of the difficulties in math (e.g., van der Sluis et al., 2005 versus Wu et al., 2008); the nature of the tasks used to measure the visual–spatial sketchpad and visual–spatial working memory; and assumptions about what various tasks measure [(e.g., Is Corsi blocks forward a measure of dynamic visual–spatial working memory or a measure of passive visual–spatial working memory?) (Passolunghi & Cornoldi, 2008)]. The range of findings is large and often inconsistent as illustrated by the following list: both static and dynamic measures have been found to differentiate children with math difficulties from typically developing peers (D'Amico & Guarnera, 2005; Reukhala, 2001); only dynamic visual–spatial working memory not static visual working memory differentiates the groups (McLean & Hitch, 1999; van der Sluis et al., 2005); even dynamic visual–spatial tasks do not differentiate the groups (Andersson & Lyxell, 2007; Bull, Johnston & Roy, 1999; Temple & Sherwood, 2002; Wu et al., 2008); and only an “active” backwards version of a dynamic visual–spatial task (Passolunghi & Cornoldi, 2008) differentiates the groups. Other studies have combined static and dynamic tasks as a measure of the visual–spatial sketchpad (Geary, Hoard, Byrd-Craven et al., 2007, 2008) and report either null findings or trends.

2.3. Conclusions

What do these studies tell us about working memory in children with math difficulties? When one considers the findings across studies, children with math difficulties differ from children without difficulties, though it could be on verbal working memory composites that sometimes contain numerical information, on static and/or dynamic visual–spatial memory tasks, on composite measures of numerical working memory, or on digit span backwards in languages with multi-syllabic number words. Although such findings are clearly of interest to the study of individual differences in math, they are not sufficient for making causal claims about relations of working memory and math, nor are the findings coherent enough to propose a model of how working memory might be implicated in mathematical disability (see Doehring, 1978 for a similar discussion of problems with univariate theories of reading disabilities). This is a question of validity — do intraindividual differences in a cognitive marker variable such as working memory help to explain math disabilities? It is important to note that low performance on a number of cognitive variables is not unexpected in children with severe learning difficulties. This has been called profile flatness (Fletcher et al., 2007) and arises from the lack of independence in the tests that are used to measure both cognitive marker variables and academic achievement. This is a question of reliability and is worth keeping in mind when interpreting putative differences in working memory between groups of children with average math achievement, vs. those with some difficulties in math achievement, vs. those with severe difficulties in math.

What are some of the barriers to understanding whether and how working memory is related to math disabilities? An understanding of which aspects of working memory are deficient in children with math difficulties is obscured by a lack of precision in knowing the particular strategies and processes that the child brings to bear on working memory tasks (possibly as a function of age and language) and a theory that links these working memory processes to particular aspects of mathematical learning and performance. There is also a lack of consistency and consensus across studies about how to measure the components of verbal and visual–spatial working memory that makes it difficult to develop a core body of knowledge on the relation of working memory and math. Some of the studies reviewed in the next section as well as the general conclusions at the end of this review deal more specifically with these issues.

3. Working memory and mathematical learning and performance: how specific are these relationships?

The reader is referred to Table 1 for summary information on several of the studies discussed in this section. Several studies examine the relationship between working memory and mathematical skills, but do so taking other academic and cognitive factors into account in order to better isolate the contribution of working memory to mathematical outcomes and mathematical development. Other studies among typically achieving individuals and children with math difficulties seek to relate working memory performance to specific mathematical outcomes or processes. Information provided by both of these approaches is useful for constructing a theory of how working memory and math are related.

Is working memory related to mathematical achievement and specific mathematical skills when other cognitive and academic factors are taken into account? Berg (2008) examined the contribution of verbal–numerical and visual–spatial working memory composites (in children in third to sixth grades) to performance on a mathematical test of a range of skills, such as single- and multi-digit arithmetic, fractions, and algebra. Verbal–numerical working memory and visual–spatial working memory contributed unique variance to mathematical performance, independent of chronological age, short-term memory, reading, and processing speed (see also Wilson & Swanson, 2001). In children with and without significant math difficulties, Swanson and Beebe-Frankenberger (2004) found that a working memory composite (verbal and visual–spatial measures) predicted solution accuracy on word problems independent of several academic and cognitive variables, such as fluid intelligence, reading skill, arithmetic achievement, knowledge of algorithms, phonological processing, short-term memory, inhibition, and so forth. In a sample of first grade children, designated as either not at risk for math difficulty or at risk and assigned to a tutor or control condition, Fuchs et al. (2005) found that a verbal working memory task (listening span) predicted end of year performance on curriculum based measures of computation, math concepts/applications, and math word problem solving (but not on an operation-specific addition fact fluency test, and a standardized calculation measure), after partialing out the influence of treatment effects and other cognitive variables, such as attention, language, nonverbal problem solving, phonological processing, and processing speed.

These findings suggest that working memory is related to a variety of mathematical outcomes when other cognitive and academic factors are taken into account, suggesting a particular role for working memory in mathematical performance. However, there also appears to be some specificity in this relation; for example, the role of verbal working memory may be greater for some aspects of math than it is for others.

Is working memory related to mathematical outcomes when domain-specific mathematical abilities are taken into account? While the above studies indicate a role for working memory in math after taking into consideration a range of cognitive and even academic achievement variables, an increasingly important question in the theoretical literature is whether domain-general abilities such as working memory significantly predict performance after taking into account domain-specific numerical abilities. One of the theoretical disputes in the math disability literature concerns whether math disabilities emerge from deficits in general purpose cognitive mechanisms such as working memory (Geary, Hoard, Nugent et al., 2007) or from deficits in domain-specific cognitive mechanisms specialized for dealing with small exact and large approximate representations of quantity, which are thought to be present, perhaps from birth, in typically developing human infants (Butterworth, 1999; Butterworth & Reigosa, 2007). Domain-specific views of math disabilities do not discount that domain-general or general purpose cognitive resources, such as working memory and language, play a role in mathematical achievement and performance, but rather argue that core deficits in these domain-specific abilities, assumed to be present early in life, have much to do with later difficulties in acquiring mathematical skills in the preschool and school-age years. Two recent studies take both domain-specific quantitative abilities and general purpose mechanisms such as working memory abilities into account in predicting mathematical development and disability (Halberda, Mazzocco, & Feigenson, 2008; Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, & Van de Rijt, 2009).

Kroesbergen et al. (2009) investigated whether domain-general (i.e. fluid intelligence, language and executive functions) and domain specific factors (subitizing — the ability to rapidly enumerate small numbers of objects without counting) predict early math skills in 5 to 7-year old children. Executive functions (which included digit span backwards as the measure of working memory updating) and subitizing explained a significant portion of the variance in children's counting skills, after controlling for language and intelligence. This study suggests that both domain general and domain specific abilities contribute to early math skills.

Halberda et al. (2008) related ninth graders' current domain specific abilities (i.e. approximate number system acuity) and previous mathematical achievement while controlling for 16 domain general abilities (general intelligence, rapid lexical access, visual–spatial reasoning, working memory, and so forth). Approximate number system acuity was assessed using a more/less judgment task. Children were presented with spatially intermixed blue and yellow dots on a computer screen for too brief a time (200 ms) to count the dots. They indicated whether there were more blue or yellow dots. Math achievement had been measured each year from kindergarten to 6th grade, and domain general abilities had been measured in grade 3. Approximate number system acuity was related to math achievement at every assessment point, and this domain specific ability retrospectively predicted the math achievement of individual students from as early as kindergarten. Moreover, the findings indicated that approximate number system acuity was related to third grade mathematical achievement even after controlling for all domain general abilities.

These studies suggest that domain specific abilities thought to be present early in life may be particularly related to mathematical achievement, and thus, highlight the importance of testing both domain specific and domain general factors for predicting math outcomes. Such studies are likely to be important for determining whether working memory not only provides a supporting role for mathematical performance, but also for whether it is causally implicated in the development of mathematical abilities and disabilities. Research that considers both domain-specific and domain-general abilities has the potential to contribute to the construction of developmental models of mathematical ability and disability.

How working memory and math are related depends on age and specific math outcomes. Studies that use working memory to predict specific mathematical outcomes and processes in children of different ages suggest that whether verbal or visual–spatial working memory is related to performance on mathematical tasks depends on age (a proxy for skill level and experience with the mathematical task) and the mathematical task itself. In preschoolers, the research of Bisanz and colleagues (Bisanz et al., 2005; Klein & Bisanz, 2000; Rasmussen & Bisanz, 2005) shows that the cognitive resources that children recruit to solve particular problems change over fairly short developmental time windows. For example, visual–spatial working memory (a preschool-friendly version of a Corsi block forward task) was the best predictor of performance for preschoolers on nonverbal addition and subtraction problems (e.g., examiner places two disks on a mat, covers the display, slides 2 more disks behind the screen and asks the child to replicate what is on the examiner's mat: a $2+2$ problem). In contrast, visual–spatial working memory did not predict performance on this same task by grade 1. Measures of phonological working memory and central executive components of working memory were the best predictors of the same problems presented verbally in the older children. These findings are consistent with Huttenlocher, Jordan, and Levine's (1994) proposal that preschoolers solve a variety of mathematical problems through the use of mental models. As language skills become stronger and verbal memory develops, children may begin to rely more on verbal memory codes to accomplish a variety of mathematical tasks including those that may have been solved using different cognitive resources at an earlier age.

Several recent studies of school age children that either contrast different age groups and/or take a wide variety of mathematical skills into account provide insight into the complexity of the relationships between working memory and math. In younger (7–8 year olds) and older (9–10 year olds) children, Holmes and Adams (2006) examined the contribution of the central executive (Listening span), visual–spatial sketchpad (a Mazes memory task), and phonological loop (non-word list recall) to achievement in a variety of mathematical domains: number and algebra (e.g. Sarah goes to the shop. She has $2.00. She spends $1.20 on a book. How much money has she got left from the $2.00?); geometry knowledge (shape, space) and measurement skills; data handling; and mental arithmetic. A cluster analysis divided the mathematical items into two discrete categories that differed depending on age: For the younger children, items fell into “pure” math (namely number and algebra, and mental arithmetic items) and “applied” math (mainly geometry and measurement and data handling items); whereas for the older children, the resulting clusters were easy and difficult items. In the younger children, the central executive, and to a lesser extent the visual–spatial sketchpad, contributed to performance on both the pure and applied areas of math. For the older children, the central executive predicted performance on both the easy and the hard items, but the phonological loop task predicted performance on the easy items, and the visual–spatial sketchpad task predicted performance on the difficult items (also see Holmes, Adams, and Hamilton, 2008). In adolescents, relations between visual–spatial working memory and math have been found (Kyttala & Lehto, 2008; Reukhala, 2001) with some differences reported for static and dynamic measures of visual–spatial working memory, depending on the particular math skill being measured (e.g., static related to mental arithmetic and dynamic related to geometry and word problem solving).

One recent study that has combined an individual difference approach with more precision over mathematical processes looked at components of working memory as *mediators* of performance on several mathematical tasks in typically developing children and in children with less and more severe difficulties in math (Geary, Hoard, Byrd-Craven et al., 2007). For children in grade 1 with more severe disabilities, tasks tapping the central executive (consisting of verbal and numerical working memory tasks) fully or partially mediated performance of this group in detecting errors on a measure of counting knowledge, retrieval errors in simple arithmetic, and accuracy on a number line estimation task. In contrast, the phonological loop or verbal memory composite (digit, word, and non-word span) and the visual–spatial memory composite (static and dynamic tasks) contributed to more specific deficits in mathematical cognition: Verbal memory was integral to counting knowledge and visual–spatial memory was important for the use of min counting while solving complex problems, to number line estimation, and to identifying and processing number-set information.

3.1. Conclusions

In general, the findings from studies of preschoolers, elementary-school children, and adolescents suggest that executive and visual–spatial skills may be recruited for the learning and application of new mathematical skills/concepts, whereas the phonological loop may come into play after a skill has been learned. This may explain the seeming paradox in the literature that it is both preschoolers and adolescents who use visual–spatial resources to accomplish developmentally appropriate mathematical tasks. In the preschool math literature, a mental models hypothesis has been applied to explain how visual–spatial memory resources are linked to mathematical learning and performance. What is unclear is whether this same explanation can be applied to new mathematical learning and/or the use of particular strategies in older children though it might help to explain some counter-intuitive findings such as why visual–spatial abilities are sometimes related to performance on math word problem solving (see Geary, 1996). These studies point to the importance of task analysis and knowledge of strategy use as a function of age and experience for both the predictors (working memory task) as well as the outcomes (particular mathematical task) when trying to understand the relation of working memory and math. Recent work looking more closely at children with math difficulties suggests that different aspects of working memory mediate different aspects of mathematical performance in severely disabled children. These findings serve to further underline a set of common themes about working memory and math that emerge from experimental, disability, and developmental studies, which will be addressed in the Conclusions and Future Directions section below.

4. Longitudinal studies of working memory and math

One type of study that may be particularly important for understanding potentially causal and/or supporting roles for working memory in mathematical cognition and development are longitudinal studies that either relate *growth in executive processes* such as working memory to math outcomes or relate early executive processes to *growth in mathematical skills*. These studies investigate specific executive processes such as updating, inhibitory control, and attention switching in relation to math, in line with models of working memory that focus on the importance of attentional control (e.g. Engle, 2002). Such longitudinal investigations provide a unique opportunity for examining the cognitive underpinnings of later developing math abilities and impairments. These studies have investigated the relation of abilities such as working memory updating and inhibitory processing (Blair & Razza, 2007; Bull, Espy, & Wiebe, 2008; English, Barnes, Taylor, & Landry, 2009; Mazzocco & Kover, 2007) as well as phonological processes (Hecht, Torgesen, Wagner, & Rashotte, 2001) to later math outcomes. Blair and Razza (2007) found that inhibitory control in preschoolers predicted kindergarten mathematical abilities, such as basic numeracy, knowledge of shapes, quantity, relative size, addition, subtraction, and simple graphic relations, as well as kindergarten reading abilities. Bull et al. (2008) found that visual–spatial working memory in the preschool years predicted math performance at the end of the third year of primary school on problems of simple and complex arithmetic, number sequencing, and graphical representation of data. In contrast, verbal working memory and inhibitory processes were related to general learning outcomes in both math and reading. Mazzocco and Kover (2007) assessed executive skills, such as fluency, inhibition, and working memory, as well as math and reading achievement in the same children at first, third and fifth grades. There were relations between executive processes and math and reading. However, those for math were dependent on age leading the authors to suggest that strong executive skills in the early primary grades may help mathematical learning and performance, but they are not necessarily associated with mathematical performance across age or across all mathematical tasks or all domains of mathematics. Hecht et al. (2001) found that latent variables representing memory processes were uniquely correlated with individual differences in a standardized estimate of the total number of mathematical computation algorithms mastered. Working with grades 2–5, Hecht and colleagues examined the unique role of latent working memory on growth in general computation skill, while controlling for prior mathematical skill, reading, speed of retrieving phonological name codes, phonological awareness, and vocabulary. Working memory was correlated with performance at each grade. An especially relevant finding was that latent memory capacity was uniquely associated with growth in mathematical computation skills while accounting for all of these variables from second- to third-grade.

4.1. Conclusions

These studies point to ways in which longitudinal information may be important for understanding growth in mathematical skills particularly when contrasted to growth in other academic domains such as reading. The findings suggest that some executive processes may be more generic in terms of supporting learning, while others, such as visual–spatial working memory may be more specific to early mathematical learning, which provides converging evidence for some of the findings from other types of studies discussed earlier. When combined with the studies from sections above, a number of possibilities suggest themselves for further study. Studies that assess how growth in components of working memory and executive processes are related to the age at which or rate at which new mathematical skills are acquired would be useful particularly when compared to the consolidation and mastery phases for those same math skills. Studies that test whether growth in different components of working memory and executive processes differentially predict growth in or level of skill in different mathematical domains such as arithmetic compared to word problem solving and geometry would begin to provide the type of information that is needed to construct a developmental model of mathematical ability and disability. Finally, studies that compare the predictive value of level and growth in general purpose cognitive mechanisms (i.e., working memory) to that provided by the integrity of domain-specific number mechanisms (i.e., subitizing and large quantity comparison) for explaining a variety of mathematical outcomes would also be useful for proposing comprehensive models of mathematical development and disability.

5. Conclusions and future directions

Research on working memory and math across experimental, math disability, and cross-sectional and longitudinal developmental studies reveal that working memory is indeed related to mathematical performance in adults and in typically developing children and in children with difficulties in math. However, they also amply demonstrate that the relations between working memory and math are complex and likely depend on several factors including, but not limited to: age, skill level, language of instruction, the way in which mathematical problems are presented, the type of mathematical skill under consideration and whether that skill is in the process of being acquired, consolidated, or mastered. The type of working memory task used and the strategies that individuals of different ages and skill levels may bring to bear in performing those tasks will also determine what one understands about working memory in relation to math. This list is perhaps a good indication that what is currently lacking in the field is a sufficiently comprehensive model of mathematical processing, particularly in relation to skill acquisition, that can handle current findings on working memory as well as provide the basis from which to guide new discoveries and inform practice. To this end, we offer suggestions about what the necessary features of such a theory might involve.

More precision in the description of math outcomes and working memory measures is important. For math, this includes obtaining some control over and knowledge of task-specific sources of variance as well as better specification of the specific math subskills that are being measured. For example, does the math skill being measured draw on procedural, conceptual, or factual mathematical knowledge (Bisanz & LeFevre, 1990). Does the domain of math matter? Do presentation variables and strategy use make a difference? Given the diverse nature of mathematical domains and skills, particularly with respect to the types of math abilities and skills that are being acquired at different ages, the use of standardized tests that measure many types of math skills together is likely to be uninformative for specifying the relation of working memory and math (Ginsburg, Klein, & Starkey, 1998). For working memory, this includes better specification of the type of memory that is measured, including the code (verbal, visual, visual–spatial and central executive), what is required to be done with the code (maintenance versus processing), and what strategies individuals bring to bear on working memory tasks. We do not see these as merely instrumentation issues; rather, we think that task analysis in both math and working memory is necessary for understanding mathematical development — ability and disability.

There is a need to account for other factors that might mediate or moderate the relations between working memory and math. Taking the research discussed above as a whole, there is evidence for interactions of child factors, such as age, math ability level, and language of instruction, and the characteristics of the mathematical tasks under consideration that may affect how working memory and math are related. However, exploration of these potential interactions are typically not included in the design of particular studies (but see Geary, Hoard, Byrd-Craven et al., 2007). Also missing from the literature is a discussion of the overlap between attention and working memory in relation to mathematics. One of the most consistent effects to have emerged in math disability research in recent years is the strong relation between math and inattention (Fuchs et al., 2006; Raghubar et al., 2009). As well, math disability and attention disorders have a high rate of co-occurrence (Fletcher, 2005; Zentall, 2007). There is considerable overlap between working memory and attention in some theoretical frameworks including developmental models of attention and executive functions (Engle, 2002; review in Garon, Bryson, & Smith, 2008) and in recent empirical studies (e.g., Alloway, Gathercole, Kirkwood, & Elliott, 2009; Liu & Tannock, 2007). This overlap suggests that the study of working memory and attention in relation to math might profit from some greater integration than has hitherto been provided in studies of the predictors of mathematical development and performance.

We have not said much in this paper about potential interactions between working memory and mathematics instruction because not much data exist to guide our thoughts (see Case & Okamoto, 1996 and Fuchs et al., 2005 for examples). In reading, the search for child cognitive characteristics by intervention interactions has proven elusive (Fletcher et al., 2007; but see Connor, Morrison, & Katch, 2004). Whether this will also be the case with math is an empirical question. It is interesting to think about what current math curricula in general education classrooms look like in this regard — they emphasize visual–spatial concepts and executive thinking abilities (Blair, Gamson, Thorne, & Baker, 2005). Children in special education may be less exposed to the teaching of these higher-level problem-solving skills (Fletcher et al., 2007). Studying child by instructional interactions in children of different ages, skill levels, and cultures that may be associated with different teaching practices, whether in general or special education, might help to inform our models of mathematical processing including the place of working memory in those models.

Decades of research on interventions to train sensory and cognitive processes, which did not directly instruct academic content, have been shown to be ineffective for improving academic outcomes (reviewed in Barnes & Fuchs, 2008; Fletcher et al., 2007). Whether cognitive training of a domain general ability such as working memory (e.g., Olesen, Westerberg, & Klingberg, 2003; Thorell, Lindqvist, Bergman, Bohlin, & Klingberg, 2008) in combination with high quality domain specific instruction in mathematics would prove to be effective particularly for younger children at risk (e.g., Barnett et al., 2008) and for older children with difficulties in math remains to be seen.

Given the importance of strategy use in mathematical processing (and in working memory tasks too) that emerges from dual task, developmental, individual difference and neuroimaging studies, the development of a theory of mathematical processing also needs to explain the links between strategy discovery, strategy selection, and execution of mathematical knowledge and specific aspects of working memory. Ideally, such a theory of mathematical processing would link the findings from cognitive, developmental, and disability studies on working memory and math with findings on how the brain processes mathematical information at different ages, ability levels, and in response to instruction.

References

Alibali, M. W., & DiRusso, A. A. (1999). The function of gesture in learning to count: More than keeping track. *Cognitive Development*, *14*, 37—56.

Alloway, T. P., Gathercole, S. E., Kirkwood, H., & Elliott, J. (2009). The cognitive and behavioral characteristics of children with low working memory. *Child Development*, *80*, 606—621.

Andersson, U., & Lyxell, B. (2007). Working memory deficit in children with mathematical difficulties: A general or specific deficit? *Journal of Experimental Child Psychology*, *96*, 197—228.

Ashcraft, M. H., Donley, R. D., Halas, M. A., & Vakali, M. (1992). Working memory, automaticity, and problem difficulty. In J. I. D. Campbell (Ed.), *The nature and origins of mathematical skills* (pp. 301—329). Amsterdam: Elsevier Science Publishers.

Baddeley, A. D. (1990). Human memory: Theory and practice US: Allyn & Bacon.

Baddeley, A. D. (1996). Exploring the central executive. *Quarterly Journal of Experimental Psychology: Human Experimental Psychology*, *49A*, 5—28.

Baddeley, A. D., & Hitch, G. J. (1974). Working memory. In G. H. Bower (Ed.), *The psychology of learning and motivation* (pp. 47—90). New York: Academic Press.

Baddeley, A. D., & Logie, R. H. (1999). Working memory: The multiple-component model. In A. Miyake & P. Shah (Eds.), *Models of working memory: Mechanisms of active maintenance and executive control* (pp. 28—61). Cambridge: Cambridge University Press.

Baker, S. C., Frith, C. D., Frackowiak, R. S. J., & Dolan, R. J. (1996). Active representation of shape and spatial location in man. *Cerebral Cortex*, *6*, 612—619.

Barnes, M. A., & Fuchs, L. S. (2008). Learning disabilities. In M. L. Wolraich, D. Drotar, & P. H. Dworkin (Eds.), *Developmental-behavioral pediatrics: Evidence and practice* (pp. 445—466). Philadelphia: Mosby.

Barnett, W. S., Kwanghee, J., Yarosz, D. J., Thomas, J., Hornbeck, A., Stechuk, R., et al. (2008). Educational effects of the Tools of the Mind curriculum: A randomized trial. *Early Childhood Research Quarterly*, *23*, 299—313.

Berch, D. B. (2008). Working memory and mathematical cognitive development: limitations of limited-capacity resource models. *Developmental Neuropsychology*, *33*, 427—446.

Berg, D. H. (2008). Working memory and arithmetic calculation in children: The contributory roles of processing speed, short-term memory, and reading. *Journal of Experimental Child Psychology*, *99*, 288—308.

Bisanz, J., & LeFevre, J. (1990). Strategic and nonstrategic processing in the development of mathematical cognition. In D. F. Bjorklund (Ed.), *Children's strategies: Contemporary views of cognitive development.* Hillsdale, NJ: Erlbaum.

Bisanz, J., Sherman, J. L., Rasmussen, C., & Ho, E. (2005). Development of arithmetic skills and knowledge in preschool children. In J. I. D. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 143—162). New York: Psychology Press.

Bjorklund, D. F. (2005). Children's thinking: Cognitive development and individual differences, 4th ed. Belmont, California: Thomson/Wadsworth.

Blair, C., Gamson, D., Thorne, S., & Baker, D. (2005). Rising mean IQ: Cognitive demand of mathematics education for young children, population exposure to formal schooling, and the neurobiology of the prefrontal cortex. *Intelligence*, *33*, 93—106.

Blair, C., & Razza, R. P. (2007). Relating effortful control, executive function, and false belief understanding to emerging math and literacy ability in kindergarten. *Child Development*, *78*, 647—663.

Bull, R., Espy, K. A., & Wiebe, S. A. (2008). Short-term memory, working memory, and executive functioning in preschoolers: Longitudinal predictors of mathematical achievement at 7 years. *Developmental Neuropsychology*, *33*, 205—228.

Bull, R., Johnston, R. S., & Roy, J. A. (1999). Exploring the roles of the visual–spatial sketch pad and central executive in children's arithmetical skills: Views from cognition and developmental neuropsychology. *Developmental Neuropsychology*, *15*, 421—442.

Butterworth, B. (1999). The mathematical brain London: Macmillan.

Butterworth, B., & Reigosa, V. (2007). Information processing deficits in dyscalculia. In D. B. Berch & M.M.M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 107—120). Baltimore, Maryland: Paul H. Brookes Publishing Co.

Campbell, J. I. D., & Clarke, J. M. (1992). Cognitive number processing: An encoding-complex perspective. In J. I. D. Campbell (Ed.), *The nature and origins of numerical skills* (pp. 457—492). Amsterdam: Elsevier Science Publishers.

Case, R., & Okamoto, Y. (1996). The role of central conceptual structures in the development of children's thought. *Monographs of the Society for Research in Child Development*, *61* v-265.

Connor, C. M., Morrison, F. J., & Katch, L. E. (2004). Beyond the reading wars: Exploring the effect of child instruction interactions on growth in early reading. *Scientific Studies of Reading*, *8*, 305—336.

Cornoldi, C., Rigoni, F., Venneri, A., & Vecchi, T. (2000). Passive and active processes in visuo-spatial memory: Double dissociation in developmental learning disabilities. *Brain and Cognition*, *43*, 117—120.

D'Amico, A., & Guarnera, M. (2005). Exploring working memory in children with low arithmetical achievement. *Learning and Individual Differences*, *15*, 189—202.

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, *44*, 1—42.

De Rammelaere, S., Stuyven, E., & Vandierendonck, A. (1999). The contribution of working memory resources in the verification of simple mental arithmetic sums. *Psychological Research*, *62*, 72—77.

De Rammelaere, S., Stuyven, E., & Vandierendonck, A. (2001). Verifying simple arithmetic sums and products: Are the phonological loop and the central executive involved? *Memory and Cognition*, *29*, 267—273.

DeStefano, D., & LeFevre, J. (2004). The role of working memory in mental arithmetic. *European Journal of Cognitive Psychology*, *16*, 353—386.

Doehring, D. G. (1978). The tangled web of behavioral research on developmental dyslexia. In A. L. Benton & D. Pearl (Eds.), *Dyslexia: An appraisal of current knowledge* (pp. 123—138). New York: Oxford University Press.

Duverne, S., Lemaire, P., & Vandierendonck, A. (2008). Do working-memory executive components mediate the effects of age on strategy selection or on strategy execution? Insights from arithmetic problem solving. *Psychological Research*, *72*, 27—38.

Engle, R. W. (2002). Working memory capacity as executive attention. *Current Directions in Psychological Science*, *11*, 19—23.

Engle, R. W., Tuholski, S. W., Laughlin, J. E., & Conway, A. R. A. (1999). The effect of memory load on negative priming: An individual differences investigation. *Memory and Cognition*, *27*, 1042—1050.

English, L. H., Barnes, M. A., Taylor, H. B., & Landry, S. H. (2009). Mathematical development in spina bifida. *Developmental Disabilities Research Reviews*, *15*, 28—34.

Fletcher, J. M. (2005). Predicting math outcomes: Reading predictors and comorbidity. *Journal of Learning Disabilities*, *38*, 308—312.

Fletcher, J. M., Lyon, G. R., Fuchs, L. S., & Barnes, M. A. (2007). Learning disabilities: From identification to intervention New York: The Guilford Press.

Fuchs, L. S., Compton, D. L., Fuchs, D., Paulsen, K., Bryant, J. D., & Hamlett, C. L. (2005). The prevention, identification, and cognitive determinants of math difficulty. *Journal of Educational Psychology*, *97*, 493—513.

Fuchs, L. S., Fuchs, D., Compton, D. L., Powell, S. R., Seethaler, P. M., Capizzi, A. M., et al. (2006). The cognitive correlates of third-grade skill in arithmetic, algorithmic computation, and arithmetic word problems. *Journal of Educational Psychology*, *98*, 29—43.

Fuchs, L. S., Fuchs, D., Stuebing, K., Fletcher, J. M., Hamlett, C. L., & Lambert, W. (2008). Problem solving and computational skill: Are they shared or distinct aspects of mathematical cognition? *Journal of Educational Psychology*, *100*, 30—47.

Furst, A. J., & Hitch, G. J. (2000). Separate roles for executive and phonological components of working memory in mental arithmetic. *Memory and Cognition*, *28*, 774—782.

Garon, N., Bryson, S. E., & Smith, I. M. (2008). Executive function in preschoolers: A review using an integrative framework. *Psychological Bulletin*, *134*, 31—60.

Geary, D. C. (1996). Sexual selection and sex differences in mathematical abilities. *Behavioral and Brain Sciences*, *19*, 229—284.

Geary, D. C., Hamson, C. O., & Hoard, M. K. (2000). Numerical and arithmetical cognition: A longitudinal study of process and concept deficits in children with learning disability. *Journal of Experimental Child Psychology*, *77*, 236—263.

Geary, D. C., & Hoard, M. K. (2005). Learning disabilities in arithmetic and mathematics. In J. I. D. Campbell (Ed.), *The handbook of mathematical cognition* (pp. 253—267). New York: Psychology Press.

Geary, D. C., Hoard, M. K., Byrd-Craven, J., & DeSoto, M. C. (2004). Strategy choices in simple and complex addition: Contributions of working memory and counting knowledge for children with mathematical disability. *Journal of Experimental Child Psychology*, *88*, 121—151.

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development*, *78*, 1343—1359.

Geary, D. C., Hoard, M. K., & Hamson, C. O. (1999). Numerical and arithmetical cognition: Patterns of functions and deficits in children at risk for a mathematical disability. *Journal of Experimental Child Psychology*, *74*, 213—239.

Geary, D. C., Hoard, M. K., Nugent, L., & Byrd-Craven, J. (2007). Strategy use, long-term memory, and working memory capacity. In D. B. Berch & M.M.M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 83—105). Baltimore, Maryland: Paul H. Brookes Publishing Co.

Geary, D. C., Hoard, M. K., Nugent, L., & Byrd-Craven, J. (2008). Development of number line representations in children with mathematical learning disability. *Developmental Neuropsychology*, *33*, 277—299.

Gerton, B. K., Brown, T. T., Meyer-Lindenberg, A., Kohn, P., Holt, J. L., Olsen, R. K., et al. (2004). Shared and distinct neurophysiological components of the digits forward and backward tasks as revealed by functional neuroimaging. *Neuropsychologia*, *42*, 1781—1787.

Ginsburg, H. P., Klein, A., & Starkey, P. (1998). The development of children's mathematical thinking: Connecting research with practice. In W. Damon, I. E. Sigel, & A. K. Renninger (Eds.), *Handbook of child psychology, 5th ed. Child psychology in practice*, *Vol. 4* (pp. 401—476). NJ: John Wiley & Sons Inc.

Halberda, J., Mazzocco, M. M. M., & Feigenson (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature*, *455*, 665—669.

Hale, S., Bronik, M. D., & Fry, A. F. (1997). Verbal and spatial working memory in school-age children: Developmental differences in susceptibility to interference. *Developmental Psychology*, *33*, 364—371.

Hecht, S. A. (2002). Counting on working memory in simple arithmetic when counting is used for problem solving. *Memory and Cognition*, *30*, 447—455.

Hecht, S. A., Torgesen, J. K., Wagner, R. K., & Rashotte, C. A. (2001). The relations between phonological processing abilities and emerging individual differences in mathematical computation skills: A longitudinal study from second to fifth grades. *Journal of Experimental Child Psychology*, *79*, 192—227.

Hitch, G. J., & McAuley, E. (1991). Working memory in children with specific arithmetical learning difficulties. *British Journal of Psychology*, *82*, 375—386.

Holmes, J., & Adams, J. W. (2006). Working memory and children's mathematical skills: Implications for mathematical development and mathematical curricula. *Educational Psychology*, *26*, 339—366.

Holmes, J., Adams, J. W., & Hamilton, C. J. (2008). The relationship between visuospatial sketchpad capacity and children's mathematical skills. *European Journal of Cognitive Psychology*, *20*, 272—289.

Hulme, C., Thomson, N., Muir, C., & Lawrence, A. (1984). Speech rate and the development of short-term memory span. *Journal of Experimental Child Psychology*, *38*, 241—253.

Huttenlocher, J., Jordan, N. C., & Levine, S. C. (1994). A mental model for early arithmetic. *Journal of Experimental Psychology: General*, *123*, 284—296.

Imbo, I., & Vandierendonck, A. (2007). The role of phonological and executive working memory resources in simple arithmetic strategies. *European Journal of Cognitive Psychology*, *19*, 910—933.

Imbo, I., & Vandierendonck, A. (2007). The development of strategy use in elementary school children: Working memory and individual differences. *Journal of Experimental Child Psychology*, *96*, 284—309.

Imbo, I., Vandierendonck, A., & De Rammelaere, S. (2007). The role of working memory in the carry operation of mental arithmetic: Number and value of the carry. *Quarterly Journal of Experimental Psychology*, *60*, 708—731.

Imbo, I., Vandierendonck, A., & Vergauwe, E. (2008). The role of working memory in carrying and borrowing. *Psychological Research*, *71*, 467—483.

Kalaman, D. A., & LeFevre, J. (2007). Working memory demands of exact and approximate addition. *European Journal of Cognitive Psychology*, *19*, 187—212.

Kintsch, W., & Greeno, J. G. (1985). Understanding and solving word arithmetic problems. *Psychological Review*, *92*, 109—129.

Klein, J. S., & Bisanz, J. (2000). Preschoolers doing arithmetic: The concepts are willing but the working memory is weak. *Canadian Journal of Experimental Psychology*, *54*, 105—115.

Knops, A., Thirion, B., Hubbard, E. M., Michel, V., & Dehaene, S. (2009). Recruitment of an area involved in eye movements during mental arithmetic. *Science*, *324*, 1583—1585.

Kroesbergen, E. H., Van Luit, J. E. H., Van Lieshout, E. C. D. M., Van Loosbroek, E., & Van de Rijt, B. A. M. (2009). Individual differences in early numeracy: The role of executive functions and subitizing. *Journal of Psychoeducational Assessment*, *27*, 226—236.

Kyttala, M., & Lehto, J. E. (2008). Some factors underlying mathematical performance: The role of visuospatial working memory and non-verbal intelligence. *European Journal of Psychology of Education*, *23*, 77—94.

Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental dyscalculia and basic numerical capacities: A study of 8–9 year-old students. *Cognition*, *93*, 99—125.

Lee, K., & Kang, S. (2002). Arithmetic operation and working memory: Differential suppression in dual tasks. *Cognition*, *83*, B63—B68.

LeFevre, J., DeStefano, D., Coleman, B., & Shanahan, T. (2005). Mathematical cognition and working memory. In J. I. D. Campbell (Ed.), *The handbook of mathematical cognition* (pp. 361—378). New York: Psychology Press.

LeFevre, J., Lei, Q., Smith-Chant, B., & Mullins, D. (2001). Multiplication by eye and by ear for Chinese-speaking and English-speaking adults. *Canadian Journal of Experimental Psychology*, *55*, 277—284.

LeFevre, J., & Liu, J. (1997). Numerical cognition: Single-digit multiplication skills of adults from China and Canada. *Mathematical Cognition*, *3*, 31—62.

Lemaire, P., Abdi, H., & Fayol, M. (1996). The role of working memory resources in simple cognitive arithmetic. *European Journal of Cognitive Psychology*, *8*, 73—103.

Li, S. -C., & Lewandowsky, S. (1995). Forward and backward recall: Different retrieval processes. *Journal of Experimental Psychology. Learning, Memory, and Cognition*, *21*, 837—847.

Liu, M., & Tannock, R. (2007). Working memory and inattentive behavior in a community sample of children. *Behavioral and Brain Functions*, *3*.

Logie, R. H., Gilhooly, K. J., & Wynn, V. (1994). Counting on working memory in arithmetic problem solving. *Memory and Cognition*, *22*, 395—410.

Mabbot, D. J., & Bisanz, J. (2008). Computational skills, working memory, and conceptual knowledge in older children with mathematics learning disabilities. *Journal of Learning Disabilities*, *41*, 15—28.

Mazzocco, M. M. M. (2007). Defining and differentiating mathematical learning disabilities and difficulties. In D. B. Berch & M.M.M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 7—27). Baltimore, Maryland: Paul H. Brookes Publishing Co.

Mazzocco, M. M., & Kover, S. T. (2007). A longitudinal assessment of executive function skills and their association with math performance. *Child Neuropsychology*, *13*, 18—45.

McCloskey, M. (1992). Cognitive mechanisms in numerical processing: Evidence from acquired dyscalculia. *Cognition*, *44*, 107—157.

McKenzie, B., Bull, R., & Gray, C. (2003). The effects of phonological and visual–spatial interference on children's arithmetical performance. *Educational and Child Psychology*, *20*, 93—108.

McLean, J. F., & Hitch, G. J. (1999). Working memory impairments in children with specific arithmetic learning difficulties. *Journal of Experimental Child Psychology*, *74*, 240—260.

Miyake, A., & Shah, P. (1999). *Models of working memory: Mechanisms of active maintenance and executive control* Cambridge, UK: Cambridge University Press.

Noel, M. -P., Desert, M., Aubrun, A., & Seron, X. (2001). Involvement of short-term memory in complex mental calculation. *Memory and Cognition*, *29*, 34—42.

Olesen, P. J., Westerberg, H., & Klingberg, T. (2003). Increased prefrontal and parietal activity after training of working memory. *Nature Neuroscience*, *7*, 75—79.

Passolunghi, M. C., & Cornoldi, C. (2008). Working memory failures in children with arithmetical difficulties. *Child Neuropsychology*, *14*, 387—400.

Passolunghi, M. C., & Siegel, L. S. (2001). Short-term memory, working memory, and inhibitory control in children with difficulties in arithmetic problem solving. *Journal of Experimental Child Psychology*, *80*, 44—57.

Passolunghi, M. C., & Siegel, L. S. (2004). Working memory and access to numerical information in children with disability in mathematics. *Journal of Experimental Child Psychology*, *88*, 348—367.

Raghubar, K. P. (2009). The role of working memory and finger representation in single-digit calculation [Abstract]. *Journal of the International Neuropsychological Society*, 15 Suppl. 1. 175.

Raghubar, K. P., Cirino, P., Barnes, M. A., Ewing-Cobbs, L., Fletcher, J., & Fuchs, L. (2009). Errors in multi-digit arithmetic and behavioral inattention in children with math difficulties. *Journal of Learning Disabilities*, *42*, 356—371.

Rasmussen, C., & Bisanz, J. (2005). Representation and working memory in early arithmetic. *Journal of Experimental Child Psychology*, *91*, 137—157.

Reukhala, M. (2001). Mathematical skills in ninth-graders: Relationship with visuo-spatial abilities and working memory. *Educational Psychology*, *21*, 387—399.

Rosen, V. M., & Engle, R. W. (1997). The role of working memory capacity in retrieval. *Journal of Experimental Psychology. General*, *126*, 211—227.

Rosselli, M., Matute, E., Pinto, N., & Ardila, A. (2006). Memory abilities in children with subtypes of dyscalculia. *Developmental Neuropsychology*, *30*, 801—818.

Rourke, B. P. (1993). Arithmetic disabilities, specific and otherwise: A neuropsychological perspective. *Journal of Learning Disabilities*, *26*, 214—226.

Seitz, K., & Schumann-Hengsteler, R. (2000). Mental multiplication and working memory. *European Journal of Cognitive Psychology*, *12*, 552—570.

Seitz, K., & Schumann-Hengsteler, R. (2002). Phonological loop and central executive processes in mental addition and multiplication. *Psychologische Beitrage*, *44*, 275—302.

Seyler, D. J., Kirk, E. P., & Ashcraft, M. H. (2003). Elementary subtraction. *Journal of Experimental Psychology. Learning, Memory, and Cognition*, *29*, 1339—1352.

Siegel, L. S., & Ryan, E. B. (1989). The development of working memory in normally achieving and subtypes of learning disabled children. *Child Development*, *60*, 973—980.

Siegler, R. S., & Booth, J. L. (2005). Development of numerical estimation: A review. In J. I. D. Campbell (Ed.), *The handbook of mathematical cognition* (pp. 197—212). New York: Psychology Press.

Simon, T. J., & Rivera, S. M. (2007). Neuroanatomical approaches to the study of mathematical ability and disability. In D. B. Berch & M.M.M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 283—305). Baltimore, Maryland: Paul H. Brookes Publishing Co.

Swanson, H. L. (2004). Working memory and phonological processing as predictors of children's mathematical problem solving at different ages. *Memory and Cognition*, *32*, 648—661.

Swanson, H. L., & Beebe-Frankenberger, M. (2004). The relationship between working memory and mathematical problem solving in children at risk and not at risk for serious math difficulties. *Journal of Educational Psychology*, *96*, 471—491.

Swanson, H. L., & Jerman, O. (2006). Math disabilities: A selective meta-analysis of the literature. *Review of Educational Research*, *76*, 249—274.

Temple, C. M., & Sherwood, S. (2002). Representation and retrieval of arithmetical facts: Developmental difficulties. *Quarterly Journal of Experimental Psychology Section A, Human Experimental Psychology*, *55A*, 733—752.

Thorell, L. B., Lindqvist, S., Bergman, S., Bohlin, G., & Klingberg, T. (2008). Training and transfer effects of executive functions in preschool children. *Developmental Science*, *11*, 969−976.

Tronsky, L. N. (2005). Strategy use, the development of automaticity, and working memory involvement in complex multiplication. *Memory & Cognition*, *33*, 927−940.

van der Sluis, S., van der Leij, A., & de Jong, P. F. (2005). Working memory in Dutch children with reading- and arithmetic-related LD. *Journal of Learning Disabilities*, *38*, 207−221.

Wilson, K. M., & Swanson, H. L. (2001). Are mathematics disabilities due to a domain-general or a domain-specific working memory deficit? *Journal of Learning Disabilities*, *34*, 237−248.

Wu, S. S., Meyer, M. L., Maeda, U., Salimpoor, V., Tomiyama, S., Geary, D. C., et al. (2008). Standardized assessment of strategy use and working memory in early mental arithmetic performance. *Developmental Neuropsychology*, *33*, 365−393.

Zentall, S. S. (2007). Math performance of students with ADHD: Cognitive and behavioral contributors and interventions. In D. B. Berch & M. M. M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 219−243). Baltimore, Maryland: Paul H. Brookes Publishing Co.

Zorzi, M., Priftis, K., & Umilta, C. (2002). Neglect disrupts the mental number line. *Nature*, *417*, 138.